Heat-Transfer Analysis of Rocket Nozzles Using Very High Temperature Propellants

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The Monte Carlo method is used as a basis for determining two-dimensional propellant temperature distributions and wall heat-transfer rates as functions of axial position in a nozzle of arbitrary shape. The propellant is considered to be at such elevated temperatures that radiation is the dominant mode of heat transfer, although the effect of convection is also considered. The propellant is assumed to be an absorbing-emitting gas with a constant absorption coefficient, and the effects of flow, variable heat-transfer coefficient, propellant heat capacity, and nozzle wall temperature are included.

Nomenclature

A_W	= area of surface of axial nozzle element, ft^2
C_p	= heat capacity of propellant, Btu/(lb)(°R)
D	= local nozzle diameter, ft
$E_{\Delta A}$	= rate of radiant energy absorption in a surface element, Btu/hr
$E_{\Delta V}$	= rate of radiant energy absorption in a volume element, Btu/hr
h	= convective heat-transfer coefficient, Btu/hr-ft^2-°R
q_{source}	= radiative energy emitted by a source in a volume element, Btu/hr
T_{in}, T_{out}	= temperature of propellant flowing into or out of a volume element, respectively, °R
$T_{\Delta V}$	= temperature of a volume element, °R
T_W	= nozzle wall temperature, °R
W	= propellant flow rate through nozzle, lb/hr
$W_{\Delta V}$	= propellant flow rate through volume element, lb/hr
ΔV	= volume of element, ft^3
$\bar{\kappa}$	= mean propellant radiation absorption coefficient, 1/ft
$\kappa_{\Delta V}$	= propellant absorption coefficient in volume element, 1/ft
σ	= Stefan-Boltzmann constant, 1.714×10^{-9} Btu/hr-ft^2-°R^4

Introduction

TYPICAL analyses of heat transfer in rocket nozzles take into account the effects of conduction and convection combined with the difficult problem of energy release by chemical reaction in the combustion chamber and nozzle. In recent concepts of rocket propulsion, such as the solid-core nuclear rocket or the gaseous-core nuclear rocket, propellant temperatures under consideration have been reaching higher and higher levels. At these temperature levels, the transport of heat by radiation becomes an important and perhaps an overriding factor in comparison to conduction or even convection.[1, 2] In nuclear rockets, combustion per se is not encountered; however, consideration of radiant heat transport in absorbing-emitting gases as an important mechanism replaces the combustion problem by one that is more difficult in many respects.

Previous workers[3, 4] have considered the effect of radiation in rocket nozzles by assuming the propellant to be transparent to radiation and then by calculating the radiant interchange between various finite segments of the nozzle wall. The effects of the nontransparency of the gas and of the interactions between the various modes of heat transfer were not taken into account. Einstein[1] and Ragsdale and Einstein[2] considered the effect of radiation in a flowing gas with constant properties, but restricted the study to a finite, right circular cylinder.

A convenient tool for attacking radiant-heat-transfer problems involving real gases in complex geometries has recently been introduced.[5] This is the use of the Monte Carlo method, which is familiar in the solution of neutron-diffusion and free-molecule gasdynamics problems. Its possible application to this type of problem was originally mentioned in Ref. 6, but has only recently been applied. Use of the Monte Carlo method makes the otherwise extremely complex problem attacked here soluble within reasonable limits of accuracy and digital computer time.

Problem

The problem analyzed is the determination of the wall heat flux as a function of axial position and the two-dimensional propellant temperature distribution in a rocket nozzle operating under steady-state conditions. The propellant mean radiation absorption coefficient is considered to be a constant, and a nozzle-wall heat-transfer coefficient is taken as an arbitrary function of axial position. The effect of propellant flow is considered, with an assumed initial slug flow profile.

Required initial conditions are the propellant mass flow rate and the temperature distribution at the nozzle inlet, the axial pressure distribution in the nozzle, and the propellant absorption coefficient and heat capacity.

Some assumptions are made that allow the neglect of conduction in the gas and along the nozzle walls, neglect of wavelength effects on radiation in the gas, and neglect of radiant emission from the nozzle walls. The basis for these and other assumptions is discussed in the analysis.

Method of Analysis

The general computer program developed for this problem uses finite-difference equations to define the magnitude of radiant energy sources in the propellant on the basis of an assumed propellant temperature profile. Another section of the program then uses these sources and the known radiant energy assumed to be entering the nozzle from the gaseous-core nuclear reactor to solve the radiation transfer by a Monte Carlo technique. From the energy emission thus determined for each gas element a new temperature profile is found. This profile is then used as a new temperature estimate, and the procedure is repeated until convergence is obtained.

In more detail, the rate of energy emission from a gas volume element ΔV, adjacent to the nozzle wall, is found by a heat balance to be

$$4\kappa_{\Delta V}\sigma T_{\Delta V}^4 = E_{\Delta V \Delta V} + W_{\Delta V}C_{p,\Delta V}(T_{in} - T_{out}) - hA_W(T_{\Delta V} - T_W) \quad (1)$$

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where $\kappa_{\Delta V}$ is the gas absorption coefficient, assumed constant throughout the element, $W_{\Delta V}$ is the mass flow rate through the element, $(C_p)_{\Delta V}$ is the propellant heat capacity, also assumed constant throughout the volume element, and T_{in} and T_{out} are the temperatures of the propellant entering and leaving the element, respectively. The first term on the right is the rate of radiant energy absorption in the element. The last two terms are, respectively, the rate of energy entering the element by flow and the rate of energy loss by convection to the wall.

The term $E_{\Delta V}$, giving the absorption of radiant energy, is the most difficult term to evaluate. It is made up of radiant energy originating in other gas elements and radiant energy entering the rocket nozzle from the reactor chamber. Small contributions to $E_{\Delta V}$ are made by radiant energy emitted by the nozzle walls; however, this portion was ignored in the analysis since it is generally negligible in comparison to other energy terms.

A Monte Carlo technique, similar to that described in Ref. 5, was used to evaluate $E_{\Delta V}$. The two sources of radiant energy in the system were assumed to be composed of bundles of finite size. These bundles were followed throughout the nozzle until final loss from the system. Each absorption of an energy bundle in a given gas element was tallied, and the energy thus absorbed made up a portion of the $E_{\Delta V}$ for that element.

The radiant sources in the gas were found by assuming that a propellant temperature drop could only be due to a loss of energy by radiation or convection from the element. Thus, the radiant source magnitude for an element is given by Eq. (1) as

$$q_{\text{source}} = W_{\Delta V}C_p(T_{\text{in}} - T_{\text{out}}) + E_{\Delta V} - hA_W(T_{\Delta V} - T_W) \tag{2}$$

With all the other terms in Eq. (1) known, $T_{\Delta V}$, the gas increment temperature, can be determined. If the element considered is not adjacent to a surface, the last (convection) term in Eqs. (1) and (2) is not present.

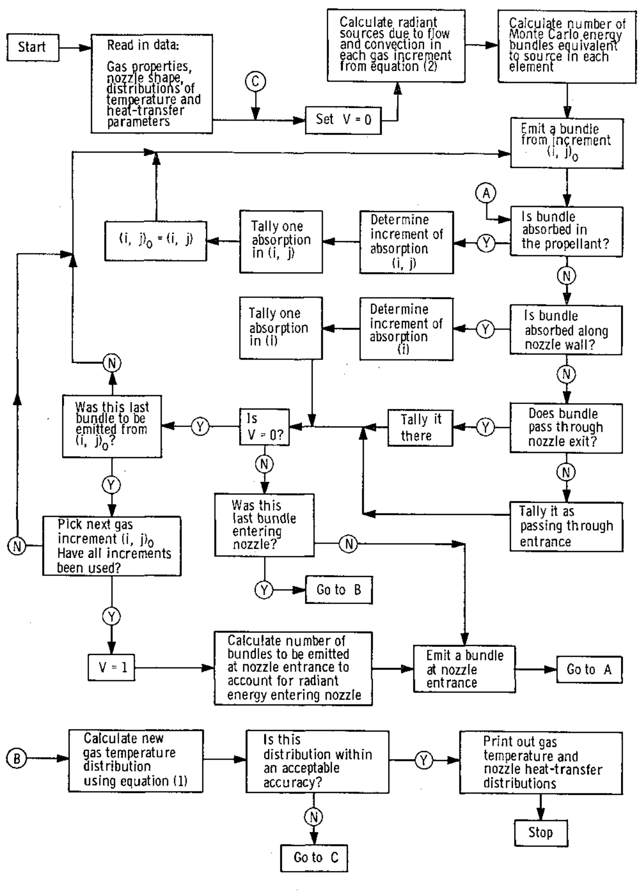

Fig. 1 Computer flow chart.

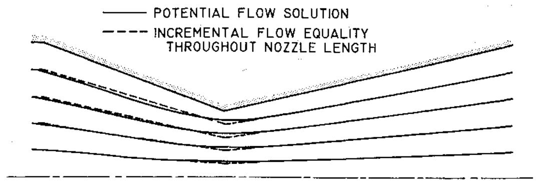

Fig. 2 Comparison of incremental flow assumption to potential flow solution.

With the temperature distribution obtained in this manner, a new set of sources is found from Eq. (2), and a new set of temperatures is computed. This procedure is followed until convergence.

To obtain the heat flux along the nozzle wall, the number of energy bundles striking the wall per unit area was simply multiplied by the energy per bundle, and this result was added to the heat flux at the wall given by a convection term. A correction for radiant emission from the wall element was then subtracted so that the total energy flux at an elemental area on the nozzle wall was given by

$$\left(\frac{q}{A_W}\right)_{\Delta V} = \frac{E_{\Delta A}}{A_W} + h(T_{\Delta V} - T_W) - \sigma T_W^4 \tag{3}$$

Computer Program

A condensed flow chart for the computer program is shown in Fig. 1 as an outline of the procedure used. The complete flow chart, including all equations, may be obtained by contacting the authors. The complete chart contains all of the equations in a step-by-step flow scheme and a list of nomenclature. The flow chart was translated for use on an IBM 7094 digital computer, as the Monte Carlo method depends on extremely fast repetitive solutions of simple equations.

The Monte Carlo method yields temperature distributions in which the individual temperatures have statistical fluctuations around a mean profile for each iteration. This leads to fluctuating heat sinks between iterations and consequent slow convergence. To avert this problem, constraints were placed on the calculated individual temperatures so that they could not exceed the peak inlet propellant temperature nor be less than the wall temperature. The guess for the temperature distribution for a new iteration was taken as an average of the distributions given by the two previous iterations. This average profile was used to calculate the radiant sources for the next iteration. The averaging procedure was similar to the introduction of a damping factor familiar in speeding the convergence of integrodifferential equations. Convergence of the temperature profiles was checked by increasing the number of energy bundles, followed by decreasing the propellant element size, and then by determining if the solution changed.

Assumptions

It was necessary to make a series of assumptions in order to somewhat simplify the problem. They are as follows.

1) No surface emission: The nozzle walls were considered to contribute no radiant energy to the system by emission. This is a reasonable assumption since the radiant energy entering the system by other means is at least an order of magnitude greater than that emitted by the cooled nozzle walls. However, a wall emission term is included in the heat balance equations used to obtain the wall heat flux.

2) Perfectly absorbing walls: An assumption of "black" walls is conservative in that interest centers on the maximum heat flux to be encountered, and any decrease in surface absorptivity will decrease the heat-transfer rates at the surface.

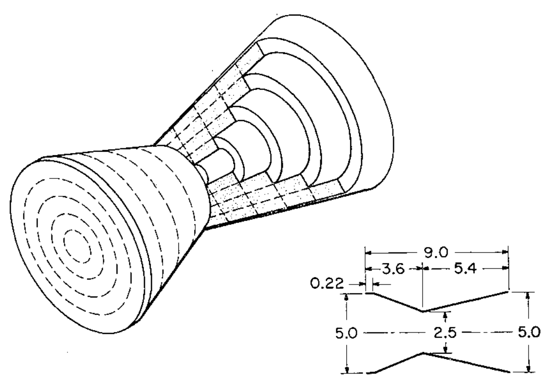

Fig. 3 Arrangement of elements and dimensions of example nozzle.

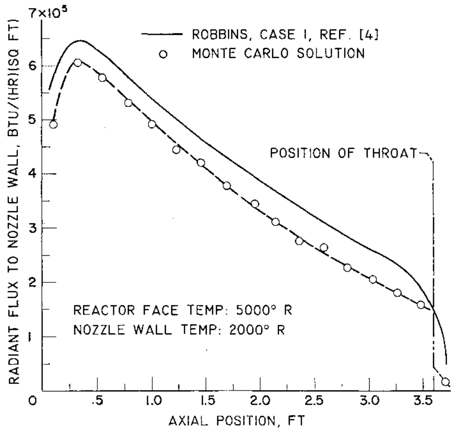

Fig. 4 Radiant heat flux to rocket nozzle walls for transparent propellant.

3) Incremental flow: The propellant entering the nozzle is assumed to maintain its entering radial mass flow distribution through the nozzle; that is, the same proportion of flow will remain in a given radial increment throughout the nozzle. This assumption was compared to a potential flow solution for the nozzle used in the example presented later (Fig. 2) and was found to be reasonable. The effects of extreme radial temperature profiles and pressure gradients on the flow distribution, however, leave the potential flow solution and therefore this assumption open to some question.

4) Gray gas: The gas absorption coefficient is assumed constant. In certain propellants, notably hydrogen, variations in the absorption coefficient are very large over the complete range of pressure, temperature, and wavelength encountered in a nozzle. However, in most nozzles, static temperature and pressure remain almost constant through the chamber, drop very rapidly through the throat, and then decrease slowly throughout the divergent section. Thus these variables do not change rapidly in the two major segments of the nozzle, and taking the propellant radiation absorption coefficient as a constant is not as severe an assumption as it appears at first glance. A further tacit assumption that the gas absorption coefficient does not vary over a mean free path is made, which in essence makes the radiation portion of this analysis a diffusion solution if variations in absorption coefficient are taken into account. Also, the absorption coefficient is assumed constant within a given gas element. Although allowance is made in the program for variation of absorption coefficient with temperature and pressure, it was taken as constant for all results presented herein.

5) No conduction: The neglect of conduction as a heat-transfer mechanism in the propellant was considered justified on the basis of the work by Einstein,[1] who showed it to be a negligible factor in similar problems. Conduction could have been considered by simply including it in Eqs. (1) and (2).

6) Axial symmetry: No circumferential variations in any physical quantities were considered.

Sample Problem

With the analysis developed in the preceding sections, the heat transfer to the nozzle walls and the gas temperature distribution for a typical case were computed. The conical nozzle shown in Fig. 3, which corresponds to a case studied by Robbins[4] for radiation through a stagnant transparent gas, was chosen, although the program will accept any nozzle shape.

Results in good agreement with those of Robbins were obtained for this limiting case, as shown in Fig. 4. The results fall somewhat below those of Robbins because the effect of radiant transfer from the nozzle wall to other elements on the wall was neglected herein.

To indicate the effect of the important parameters, this same nozzle shape was studied under simplified sample conditions. The input conditions used are representative of those expected in nozzles used in conjunction with gaseous core nuclear reactors.[7]

The propellant is assumed to enter the nozzle at a constant bulk temperature of 13,000° R in a slug flow profile. The nozzle wall is taken to be at a temperature of 5000° R.

The first variable studied is the propellant radiation absorption coefficient $\bar{\kappa}$. This parameter is assumed constant in the example, although, as mentioned previously, the program will accept it as a function of local propellant temperature and pressure. Figure 5 shows its effect on the heat transfer to the nozzle wall. As $\bar{\kappa}$ is increased and the propellant becomes more optically dense, the peak heat flux increases and moves closer to the nozzle inlet. This effect is due to the strong absorption and re-emission of radiant energy in the optically dense propellant at the entrance to the nozzle which traps energy that would otherwise pass through and be absorbed at the nozzle wall downstream.

Figure 6 shows the propellant temperature profile near the nozzle wall as flow and/or heat capacity of the propellant are increased. This effect is shown by increasing the parameter WC_p, the product of total flow rate and propellant heat capacity. The over-all propellant temperature increases as WC_p is increased, since a smaller proportion of the propellant enthalpy is lost to the nozzle surface.

In Fig. 7 the change in radiant heat-transfer rate to the nozzle wall with increasing WC_p is demonstrated. The over-all level increases rapidly as WC_p is raised. As WC_p reaches very high values, the effect of flow predominates over the

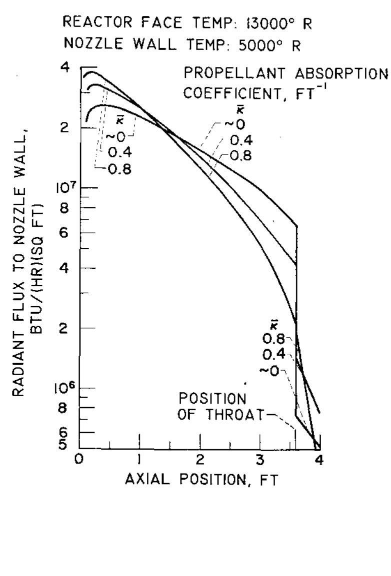

Fig. 5 Effect of propellant absorption coefficient on nozzle-wall heat flux: no flow.

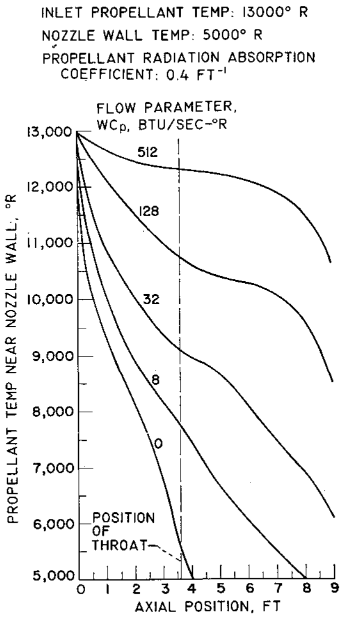

Fig. 6 Effect of flow parameter on propellant temperature.

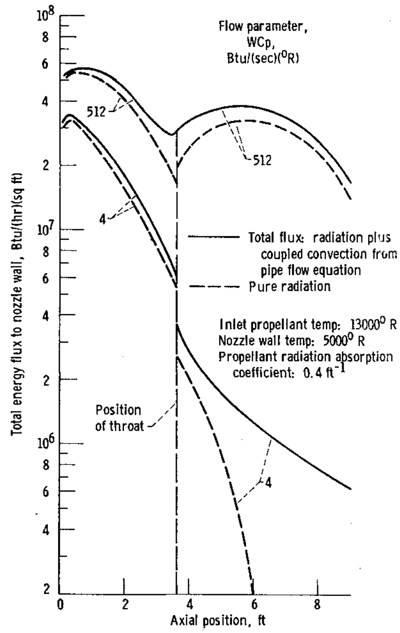

Fig. 8 Effect of convection on total heat flux.

radiant energy entering the upstream end of the nozzle. The high flow rate, combined with the diverging geometrical shape past the nozzle throat, causes a second peak to occur in the heat flux. This is because, at the throat, an area element on the nozzle wall sees a relatively thin layer of hot propellant and a large expanse of cooled wall. As the nozzle diverges, a wall element sees more hot propellant. Near the nozzle exit, an element sees hot propellant and also the cold environment outside the nozzle, causing the net radiant flux to decrease.

Figure 8 compares the nozzle-wall heat flux at a high and a low WC_p before and after the addition of a typical convective heat-transfer coefficient given by the pipe flow equation as described by Bartz[8] for similar cases. Propellant properties were taken as those of hydrogen evaluated at 100 atm and 13,000°R.

It should be noted that for hydrogen the value of WC_p should be about $940 \times 12 = 11{,}280$(Btu/sec-°R) to produce a Mach number of one at the throat of this nozzle. Thus the curves presented, if used for hydrogen, would be valid only during start-up of the propulsion system. At high values of (WC_p), convection would become relatively much more important, because the radiation flux reaches a maximum level when the propellant temperature becomes nearly constant through the nozzle. The convection, however, is strongly dependent on the mass flow rate and would reach much larger values than those implied by Fig. 8.

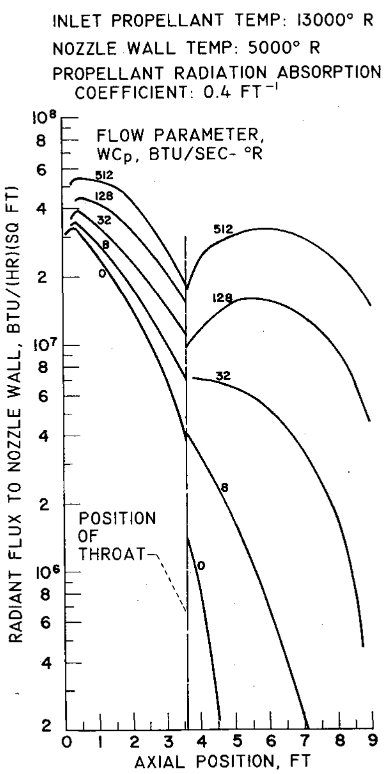

Fig. 7 Effect of flow parameter on radiant energy flux to nozzle wall.

For other nozzle geometries, it is difficult to predict effects on radiation because of the coupling present in Eq. (1) between flow and convection. However, some general effects which are expected are the following.

For a more sharply converging nozzle (smaller throat diameter), the radiant flux in the converging portion would be larger, since the wall area is exposed to the reactor chamber radiation at an angle near the normal. For a nozzle with a larger diameter, the radiant flux would increase because any wall element sees a larger volume of radiating gas. (This statement is valid only up to the nozzle diameter at which the gas opacity becomes so large that a wall element sees only radiating gas and is no longer affected by other wall elements.)

Concluding Remarks

A method of analysis suitable for a group of nozzle-heat-transfer problems in which the propellant enters at very high temperatures, so that radiation energy transfer is important, is presented. For a simplified sample problem it is shown that the portion of total energy transfer to the nozzle wall due to radiation could outweigh that due to convection. Also, the peak flux could occur near the nozzle entrance early in the convergent portion rather than near the throat for the gray propellant assumed in the example. Both of these results are in sharp contrast to those found in chemical rockets, where convective heat transfer predominates.

The level of heat fluxes encountered in the example, especially near the nozzle inlet, indicates that serious nozzle-cooling problems may be encountered for mean propellant temperatures in the 10,000° to 15,000°R range, even if maximum temperatures exist only along the axis of the nozzle-flow passage.

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Dynamic Response of a Long Case-Bonded Viscoelastic Cylinder

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Time-dependent pressures are applied in an encased viscoelastic cylinder and on the surface of the case. The resulting dynamic response of the cylinder-case system is the subject of a theoretical analysis. The viscoelastic material of the cylinder is assumed to be incompressible and a forced vibration is therefore excited without initial wave effects. The cylinder is viscoelastic in shear, showing short-time elastic behavior and delayed elasticity. The displacement, the circumferential stress, and the radial stress are investigated. If a time-dependent pressure is applied to the case, the radial bond stress at the cylinder-case interface is periodic and shows tensile peaks for high values ($\sim 10^4$) of the ratio of Young's modulus of the shell to the rubbery shear modulus of the cylinder. The stresses are damped exponentially to the compressive quasi-static solution. Analytical solutions are presented for step loading and standard linear viscoelastic shear behavior. A numerical procedure using measured values of the relaxation function is indicated. The solutions are relevant to a compressible cylinder for times long compared to the passage-time of a dilatational wave.

Introduction

DESIGN considerations for solid propellant rocket motors have led to detailed investigations of a structural system consisting of a hollow viscoelastic cylinder which is contained in a thin elastic case (shell). A number of problems have been formulated and solved for the axially symmetric plane-strain configuration. Quasi-static linear viscoelasticity theory has generally been used.

The quasi-static response of a cylinder-shell system to a time-dependent internal pressure is discussed by Bland.[1] Williams et al.[2] have also surveyed several problems of this type. A short review of the cylinder problem for quasi-static viscoelasticity is included in a recent paper by Rogers and Lee.[3]

Until quite recently, dynamic effects were not taken into account in determining the viscoelastic cylinder response. Emphasis was placed rather on improving the material specification and on including the effect of an ablating inner surface. Increasing importance of the vibrations of solid propellant rocket motors has, however, necessitated research in the area of vibrating viscoelastic cylinders.

In the present paper a long encased cylinder is subjected to time-dependent pressures at the inner and outer surfaces, inertia being taken into account. The displacements, the circumferential stresses, and the radial stresses are investigated. Of particular interest is the radial stress at the interface between the cylinder and the shell. The influence of structural and material parameters on the radial bond stress has been given special attention.

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The sudden application of a constant pressure in a cylindrical cavity generates a compressional wave. The propagation of such a wave in an infinite medium has been investigated by Selberg.[4] If a time-dependent pressure is applied in an encased cylinder, the compressional wave interacts with the case and the circular inner boundary. Since the dilatational wave velocity is very high, it is to be expected that a steady-state forced vibration is established in the cylinder-case system within a very short time.

In this paper, the viscoelastic material of the cylinder is assumed to be incompressible. For dynamic problems the assumption of incompressibility implies an infinite dilatational wave velocity. The application of pressure in an incompressible viscoelastic cylinder therefore initiates a damped vibration without initial wave effects.

The assumption of incompressibility is based on observed mechanical behavior of solid propellant binders. These rubbery materials are often considered as essentially incompressible. No real material can, however, be completely incompressible. To appraise the value of the present solutions we have to contrast them with quasi-static solutions, and with the transient solution that was described earlier. For a compressible cylinder the validity of quasi-static solutions requires small changes of stress to occur in times much larger than the wave propagation time across the body. Thus, the solutions for a suddenly applied pressure must be interpreted as having meaning only for such a ramp loading, and they should not be interpreted strictly as the response to a mathematical step function. It is evident therefore that, for a suddenly applied pressure and for the geometry which is considered in this paper, a dynamic solution under the assumption of incompressibility is superior to a quasi-static solution. The possibly important initial wave effects which occur in a compressible cylinder are not included, however, and the present solution